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The influence of incoherent co-tunnelling on single-electron-tunnelling thermometry

Heinz-Olaf Müller†, Ulrik Hanke and K A Chao

Department of Physics, Norwegian University of Science and Technology, Norwegian Institute of Technology, N-7034 Trondheim, Norway

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Abstract. Incoherent co-tunnelling in metallic ultrasmall-tunnel-junction systems is studied at high temperature, $k_BT \gtrsim E_C$, in a constant-lifetime approach involving intermediate virtual states. For the double junction the resulting conductance increases linearly with the temperature. For the serial array a sharp drop off of the conductance with the number of junctions in the array is found.

1. Introduction

The classical temperature range of single-electron tunnelling (SET) is $k_BT \ll E_C$ where E_C is the Coulomb energy. However, one of the earliest SET applications, the thermometer [1], is operated at much higher temperature, $k_BT \gtrsim E_C$, which is also the temperature domain of this study.

SET thermometry takes advantage of the temperature dependence of the Coulomb blockade in ultrasmall-tunnel-junction systems by measuring their conductance. Therefore this kind of thermometer measures the temperature of the electron system, not the temperature of the lattice. In the mK region these values might differ from each other due to the heating of the tunnelling charges themselves (the hot-electron effect [2]). Therefore it turns out to be advantageous to operate a SET thermometer at low bias voltage, i.e. to determine the temperature from the zero-bias conductance $G(V = 0, T)$. The junction systems used or proposed for SET thermometry so far are serial arrays of at least two junctions.

Incoherent co-tunnelling is the enhancement of the tunnelling current due to tunnelling events via virtual intermediate charge states, thus resulting in a distortion of the thermometer operation. It was studied first theoretically in reference [3], and this study was followed by numerous theoretical calculations [4–6] and experiments [7–9], to mention but a few early works on this subject. The vast majority of the papers on this topic, however, deal with co-tunnelling at low temperature, i.e. in the classical temperature range of SET. To the best of our knowledge only one paper [10] explicitly addresses the temperature range considered here, but with different conclusions. These differences will be discussed later on.

In the low-temperature range the zero-bias conductance $G^{\rm in}(V = 0, T)$ due to incoherent co-tunnelling is found to obey

$$G^{\rm in}(V=0,T) = \frac{\pi}{3}\,\frac{\hbar/e^2}{R^2}\left(\frac{\beta E_C}{2}\right)^{-2} \qquad (1)$$

† E-mail: hom@phys.unit.no.

for a symmetric double junction [4]. E_C, the Coulomb energy, is in this case $e^2/(4C)$, and $\beta = 1/(k_B T)$ is used for brevity. For the serial array each additional junction adds a factor $(\beta E_C)^{-2}$ [11]. In contrast, for very high temperature, ohmic behaviour of all of the junctions under consideration is expected [5], since thermal fluctuations will conceal the charging effect. With regard to SET thermometry, however, co-tunnelling is still of interest since it is a limiting factor for the thermometer's resolution.

Owing to singular matrix elements, the formal evaluation of incoherent co-tunnelling is not straightforward. Different procedures for overcoming this problem have been introduced in the past, namely 'leading-log' regularization [5, 12], Green's functions techniques [13, 14], and the simple finite-lifetime approach [15, 16]. Since the concern of this paper is a first-order estimation, the latter method is used, but the discussion will include consideration of the other procedures.

For the system under consideration, we assume the weak-tunnelling regime, i.e. $R_i \gg \hbar/e^2$, to be appropriate. Therefore quantum fluctuations are suppressed by high tunnel resistances R_i, and second-order perturbation theory [4] can be used. Furthermore, coherent co-tunnelling is not considered in our study, since it can effectively be controlled by the system layout. This is also the case for higher-order tunnel processes [14], because of their small influence.

Incoherent co-tunnelling in ultrasmall double junctions is considered in section 2, whereas section 3 concerns the longer 1D array.

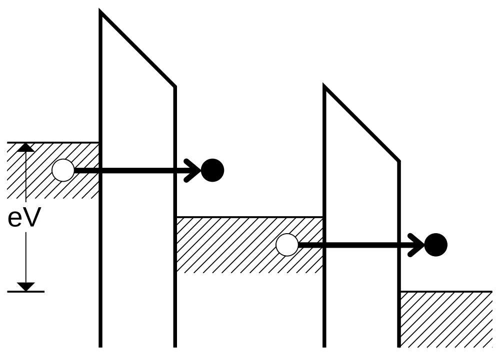

Figure 1. Incoherent co-tunnelling through a double junction. Two different charges are involved in two tunnel events leaving an electron–hole excitation behind on the island between the junctions.

2. Co-tunnelling in double junctions

The process of incoherent co-tunnelling through a double junction is shown schematically in figure 1. Two different (incoherent) charges are involved in two tunnel events, one at each junction.

The tunnel rate $\gamma(V)$ of this process follows from second-order perturbation expansion [4]:

$$\gamma(V) = \frac{\hbar}{2\pi e^4 R_1 R_2} \int d\varepsilon_1\ d\varepsilon_2\ d\varepsilon_3\ d\varepsilon_4\ f(\varepsilon_1) f(-\varepsilon_2) f(\varepsilon_3) f(-\varepsilon_4) M^\star M$$

$$\times\ \delta(eV + \varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4) \tag{2}$$

where the energy-dependent part of the matrix element, M, is given by

$$M = \frac{1}{\varepsilon_2 - \varepsilon_1 + E_1(V)} + \frac{1}{\varepsilon_4 - \varepsilon_3 + E_2(V)} \tag{3}$$

and $f(\varepsilon)$ denotes the Fermi function. Here the energies $E_{1,2}$ describe the energies necessary to transfer a charge across the respective junctions. They depend on the applied bias voltage V, the number n of excess charges on the island, and the influence of a gate possibly coupled to the island. The influence of the gate is not considered here. The applied voltage V itself is thought to be small ($eV \ll E_C$). Hence, there is no excess charge on the island ($n = 0$; the influence of the temperature on the excess charge distribution of the island is dealt with in the appendix). The energies $E_{1,2}$ are then [4]

$$\begin{aligned} E_1(V) &= E_C\left(1 - \frac{2VC_2}{e}\right) \\ E_2(V) &= E_C\left(1 - \frac{2VC_1}{e}\right) \end{aligned} \tag{4}$$

where $E_C = e^2/(2C_1 + 2C_2)$ is the Coulomb energy of the system and $C_{1,2}$ are the capacitances of the respective junctions. In the small-bias region considered, $E_{1,2}(V) > 0$.

Whereas at $T = 0$ the quadratic singularities of (2) are outside the integration range for voltages below the blockade threshold, a regularization of these singularities is required at $T > 0$. This is done by introducing a small energy $\Gamma < E_{1,2}$ into the matrix element M [15]:

$$M = \frac{1}{\varepsilon_2 - \varepsilon_1 + E_1 + \mathrm{i}\Gamma} + \frac{1}{\varepsilon_4 - \varepsilon_3 + E_2 + \mathrm{i}\Gamma}$$

resulting in

$$\begin{aligned} M^\star M = {} & \frac{1}{(\varepsilon_2 - \varepsilon_1 + E_1)^2 + \Gamma^2} + \frac{1}{(\varepsilon_4 - \varepsilon_3 + E_2)^2 + \Gamma^2} \\ & + \frac{2[(\varepsilon_2 - \varepsilon_1 + E_1)(\varepsilon_4 - \varepsilon_3 + E_2) + \Gamma^2]}{[(\varepsilon_2 - \varepsilon_1 + E_1)^2 + \Gamma^2][(\varepsilon_4 - \varepsilon_3 + E_2)^2 + \Gamma^2]}. \end{aligned} \tag{5}$$

This approximation make sense in the case where $\Gamma < E_C$, when the finite lifetime of the intermediate charge states is still large on the time-scale of $\hbar/E_C$. Otherwise, more difficult regularization procedures [5, 13] are necessarily required.

Additional simplification is achieved by neglecting the third term in (5). As $\Gamma \ll E_{1,2}$ holds, this term will not result in a significant contribution to the rate γ or the conductance G^{in}. Furthermore, the evaluation of the integrals in (2) is simplified by the peaked, temperature-independent matrix elements in comparison to the smooth Fermi functions in the temperature range considered: the latter is assumed to be constant with regard to the integration, using the peak value of the former in its argument.

Otherwise, the evaluation of the integrals is straightforward, and we find for the zero-bias conductance

$$G^{\mathrm{in}}(V = 0, T) = 2e\gamma'(V = 0) = \frac{\hbar/e^2}{R_1 R_2}\,\frac{E_C}{2\Gamma}\,\frac{\beta E_C/2}{\sinh^2(\beta E_C/2)}. \tag{6}$$

This is our main result and covers temperatures $k_B T \gg \Gamma$. Hence, if $\Gamma \ll E_C$ holds, equation (6) is valid for temperatures $k_B T \leqslant E_C$ as well.

Equation (6) is independent of an asymmetry in the junction parameters as long as $R_1 R_2$ and $C_1 + C_2$ are conserved, which coincides with the low-temperature result [4]. On the other hand, the low-temperature approximation yields $G^{\text{in}}(V = 0, T) \propto T^2$, whereas (6) gives an exponential drop off in this limit.

The behaviour of (6) depends strongly on the value of Γ, which is proportional to the inverse lifetime of a virtual state on the island. It is affected by different processes like other tunnelling processes (of first or higher than second order) or electron–electron scattering. The relevant process is not easy to spot. We derive a rough estimation of Γ from the following argument: since the energy scale of the problem is the Coulomb energy E_C, the crossover from low to high temperature takes place in the vicinity of $\beta E_C = 1$. A comparison of formulae (1) and (6) yields

$$\Gamma \approx \frac{3E_C}{4\pi} < E_C. \tag{7}$$

Γ is taken to be temperature independent, assuming a variation of it on an energy scale other than E_C. Since the following discussion of its value reveals a certain variety of the data in literature (within one order of magnitude), this crude assumption appears to be adequate for our approach. In reference [5] the peak conductance is found to be

$$G^{\text{in}}_{\text{GM}} = \frac{2\pi e^2}{\hbar} \frac{R_1 R_2}{(R_1 + R_2)^2}.$$

In comparison to our result for $G^{\text{in}}(V = e/(C_1 + C_2))$ in the limit of low temperature, $\hbar E_C/(2e^2 R_1 R_2 \Gamma)$, we see that our value (7) is for a typical set-up of the right order of magnitude, but quite large. A similar result is found in reference [13], $\Gamma_{\text{LE}} = \hbar E_C/(2e^2 R_{1,2})$, whereas a comparison with reference [15] yields reasonable correspondence, $\Gamma_{\text{KH}} = 4\pi^2 \hbar E_C/(e^2 R_{1,2})$. With regard to the experiment we conclude a conductance $G^{\text{in}}_{\text{EZ}} \approx 0.06\ \mu$S from reference [9]. In order to fit this value, Γ should be larger than the value of (7).

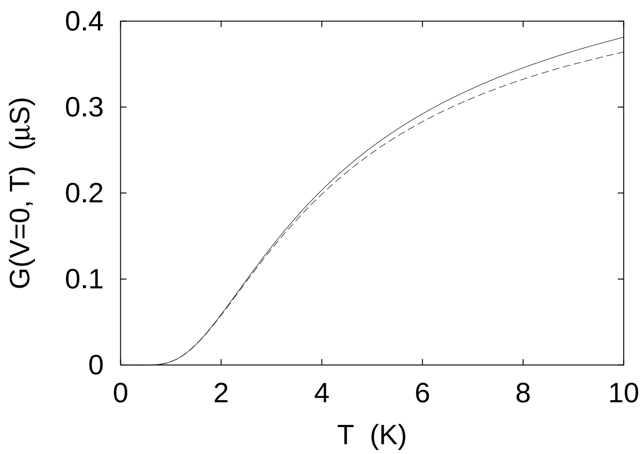

Figure 2. The dependence of the zero-bias conductance of a symmetric double junction ($C = 0.1$ fF, $R = 1$ MΩ) on the temperature. The solid line shows the full result. For the dashed curve, co-tunnelling is neglected.

In figure 2 the impact of (6) on the overall conductance of a double junction is shown with Γ corresponding to (7). At high temperature (compared to E_C/k_B) the conductance is found to increase linearly:

$$G^{\text{in}}(V=0,T) \xrightarrow{\beta\to 0} \frac{\hbar/e^2}{R_1R_2}\,\frac{k_BT}{\Gamma}. \tag{8}$$

Owing to the value of Γ this conductance is usually below the ohmic first-order conductance $1/(R_1+R_2)$. At very high temperature (several times E_C/k_B), however, it will equal the first-order conductance, and the applied perturbation expansion fails. In fact, a transition to ohmic conductance is expected in this situation [5].

In experiment, incoherent co-tunnelling has only been observed at low bias voltage and at low temperature so far, where the first-order current is suppressed [7–9]. At higher temperature, measurements have to separate the small co-tunnelling contribution from the dominating first-order current, which in turn complicates the experiment. Therefore, verification of (6) in experiment is difficult to achieve.

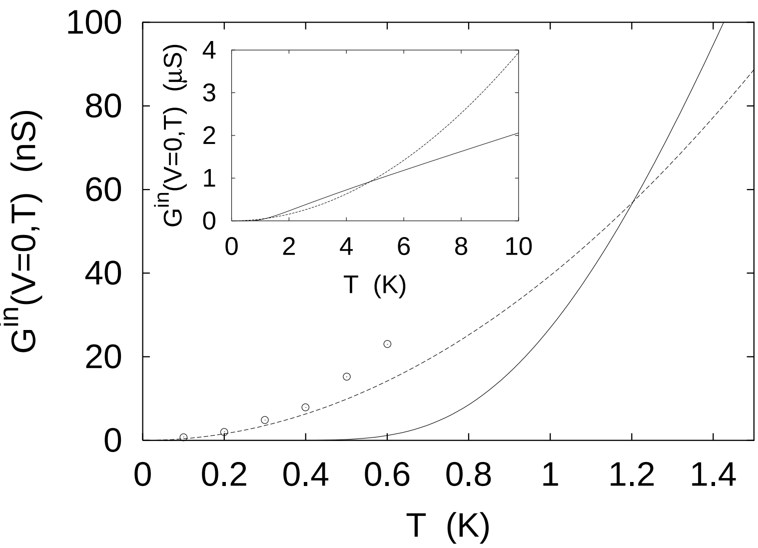

Figure 3. The dependence of the zero-bias conductance $G^{\text{in}}(V=0,T)$ of a double junction on the temperature. The symbols (⊙) display the experimental data of reference [9]. Dashed and full lines represent the corresponding low- (1) and high- (6) temperature approximations, respectively. The inset shows both formulae on a larger scale.

The temperature dependence of the zero-bias conductance in the low-temperature domain was measured in [9]. In figure 3 the data from this measurement are compared with the low-temperature approximation (1) and the high-temperature formula (6). Even if the latter gives a smaller slope on a large scale, it dominates the former within an intermediate-temperature domain. While far from being conclusive, the experimental data suggest an additional conductance contribution in this range, thus supporting the approach presented.

In contrast, in reference [10] singular matrix elements (without regularization) lead to a significant conductance via incoherent co-tunnelling at high temperature. This behaviour was not found in experiment so far. Additionally, we share the point of view of papers on co-tunnelling at low temperature [5, 12–15] that these singularities are artefacts of the perturbation theory applied.

3. Co-tunnelling in junction arrays

The investigation of incoherent co-tunnelling in the case of serial arrays of ultrasmall tunnel junctions is interesting at high temperature as well. While at low temperature the suppression of co-tunnelling by means of the array was shown to follow [11]

$$\left(\frac{R_Q}{R}\,\frac{(eV)^2+(k_BT)^2}{(2\pi E_C)^2}\right)^N$$

where N is the number of junctions in the array, it remains unclear whether a similar expression holds for high temperature. In order to clarify the behaviour of co-tunnelling, the consideration of an array of identical junctions is expected to be sufficient. Then, the second-order tunnelling matrix element is [11]

$$M_N=\sum_{j=1}^{N}\left(\prod_{\substack{i=1\\ i\neq j}}^{N}\frac{1}{\varepsilon_{2i}-\varepsilon_{2i-1}+E_i(j)}\right).$$

Like in the case of the double junction, one has to regularize the quadratic singularities of these matrix elements for non-vanishing temperatures:

$$\begin{aligned}\gamma_N(V)\propto\int &\mathrm{d}\varepsilon_1\ \mathrm{d}\varepsilon_2\ \mathrm{d}\varepsilon_3\ \cdots\ \mathrm{d}\varepsilon_{2N}\ f(\varepsilon_1)f(-\varepsilon_2)f(\varepsilon_3)\cdots f(-\varepsilon_{2N})\\ &\times\delta(eV+\varepsilon_1-\varepsilon_2+\varepsilon_3-+\cdots-\varepsilon_{2N})\left(\sum_{i=1}^{N}[(\varepsilon_{2i}-\varepsilon_{2i-1}+E_i)^2+\Gamma^2]\right)\\ &\times\left(\prod_{i=1}^{N}[(\varepsilon_{2i}-\varepsilon_{2i-1}+E_i)^2+\Gamma^2]\right)^{-1}.\end{aligned}\tag{9}$$

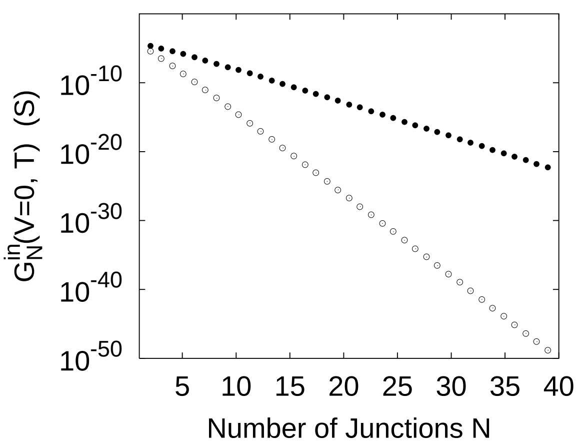

Figure 4. The dependence of the zero-bias conductance $G_N^{\mathrm{in}}(V=0)$ of a 1D array of identical junctions ($C=0.2$ fF, $R=30$ kΩ) on the number N of junctions in the array for $T=1$ K ($\odot$) and $T=5$ K (●).

Now, the evaluation follows exactly the pattern of the double junction and, generalizing equation (8),

$$G_N^{\mathrm{in}}(V=0,T)\xrightarrow{\beta\to 0}\frac{N}{R}\left(\frac{\hbar/e^2}{2R\beta\Gamma}\right)^{N-1}\tag{10}$$

is found to be the high-temperature approximation. For the double junction the known result is recovered. Within the range of validity, equation (10), as shown in figure 4, reveals a drastic suppression with increasing number of junctions in the array. For large N an almost exponential drop off of the conductance is expected. Hence, the suppression of co-tunnelling by means of longer serial arrays of ultrasmall tunnel junctions works well at high temperature also.

The dependence of the conductance (10) on the length of an array of ultrasmall tunnel junctions is—due to the rapid drop off—difficult to measure in experiment. However, in [7] data for arrays of two and three junctions are given. At the temperature of the experiment ($T \leqslant 50$ mK) the conductance ratio G_2/G_3 is about 200. In terms of (10) our formula yields $G_2/G_3 \approx 60$. Since the temperature of the experiments differs from the domain considered here, the rough correspondence in order of magnitude is satisfactory.

4. Conclusion

In conclusion, our approach yields a description of incoherent co-tunnelling at small bias voltages but high temperatures on the scale of the Coulomb energy. The perturbation expansion that this approach is based on yields singularities of the matrix elements used. Their regularization is achieved using a finite lifetime of the intermediate virtual states. This lifetime is taken to be constant over the temperature range considered. For sufficiently large junction resistances (in comparison to $\hbar/e^2$) the conductance contribution of incoherent co-tunnelling turns out to be small. At very high temperature, $T \gg E_C/k_B$, a crossover to ohmic behaviour is expected which is not described by our approach.

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Appendix

For an arbitrary number n of excess island charges the energies (4) generalize to

$$E_1(n, V) = E_C\left(1 + 2n - \frac{2VC_2}{e}\right)$$

$$E_2(n, V) = E_C\left(1 - 2n - \frac{2VC_1}{e}\right)$$

resulting in a current contribution $I(n, V, T)$ similar to the integral evaluated above in the case where $n = 0$ (section 2). Now the total current that has to be considered for further analysis is

$$I^{\text{in}}(V, T) = \sum_n I^{\text{in}}(n, V, T)\,\sigma(n, V, T)$$

where $\sigma(n, V, T)$ is the temperature-dependent occupation probability of n charges. At voltages $eV \ll k_B T$ this probability becomes independent of the bias voltage V [17]:

$$\sigma(n, T) = \sqrt{\frac{\beta E_C}{\pi}}\ \exp(-n^2 \beta E_C). \tag{A1}$$

Hence, the zero-bias conductance simplifies to

$$G^{\rm in}(V=0,T)=\sum_n G^{\rm in}(n,0,T)\sigma(n,T)$$

using $G^{\rm in}(n,V,T)=\partial I^{\rm in}(n,V,T)/\partial V$. The general expression for $G^{\rm in}(n,0,T)$ is rather awkward, so we restrict ourselves to the case of high temperature once more. In this limit one finds

$$G^{\rm in}(n,0,T)\xrightarrow{\beta\to 0}\frac{\hbar/e^2}{R_1R_2}\,\frac{k_{\rm B}T}{\Gamma}\left[1-\frac{(\beta E_{\rm C})^2}{12}(1+4n^2)\right].$$

For $|n|\ll 3/(\beta E_{\rm C})$ this result is almost independent of n. According to (A1) it becomes obvious that this range includes all significantly occupied charge states. Hence, the approximation

$$G^{\rm in}(V=0,T)=\sum_n G^{\rm in}(n,0,T)\sigma(n,V)\approx G^{\rm in}(0,0,T)\sum_n\sigma(n)=G^{\rm in}(0,0,T)$$

is useful.

Note added in proof. A qualitative discussion of co-tunnelling in complex circuits at high temperature provided by a recent paper [18] leads to conclusions similar to ours.

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